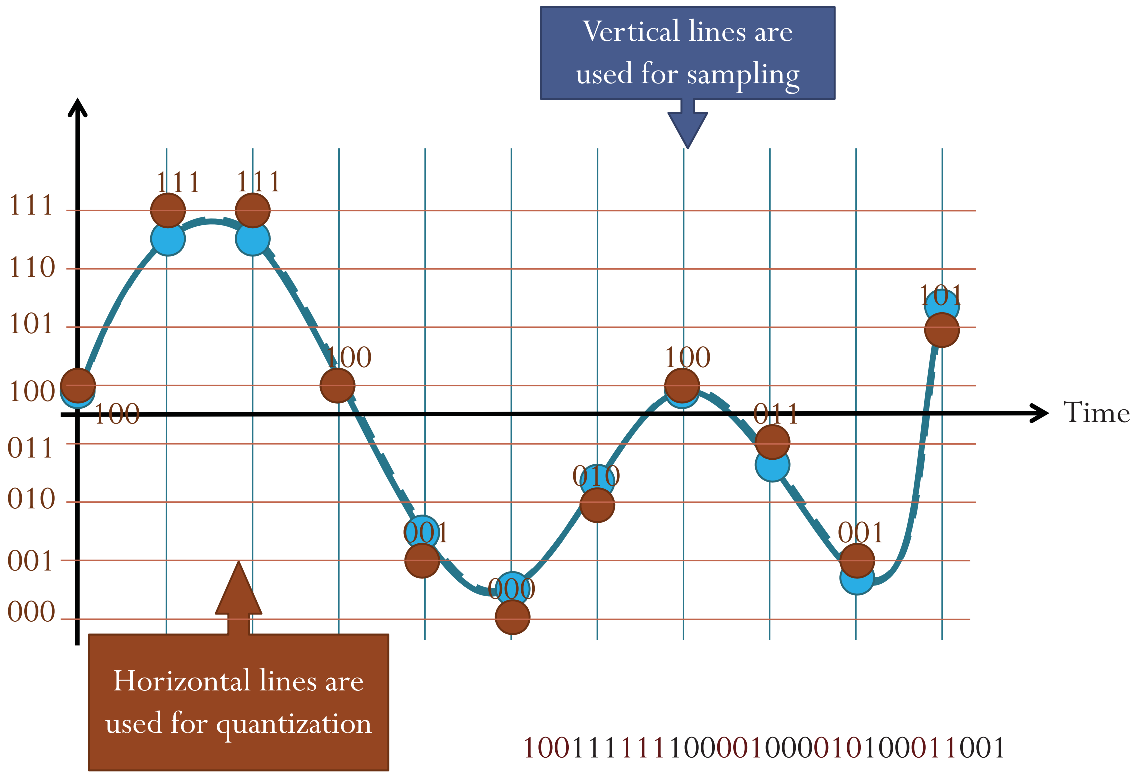
Fourier Transform and Communication Systems

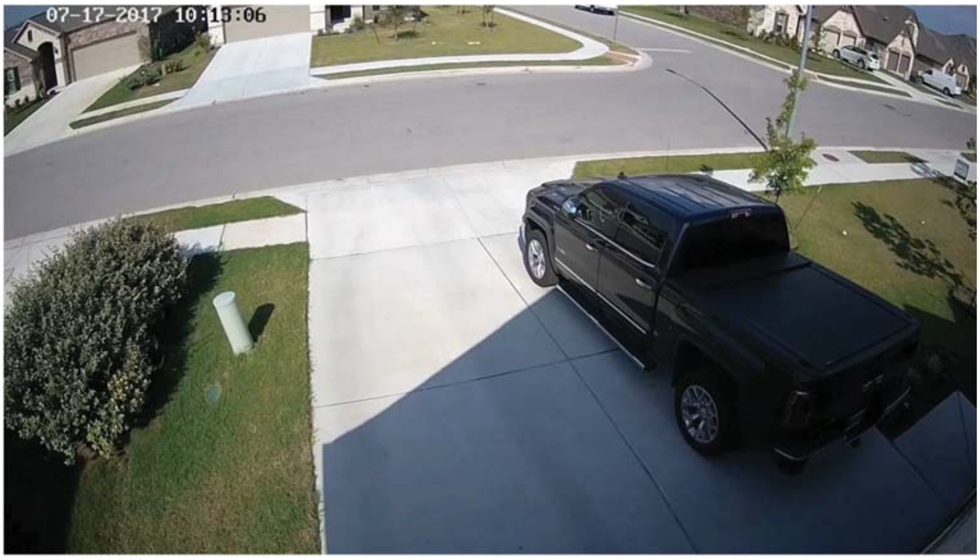
Sampling: Effect in the Frequency Domain

Digitization (analog to digital)



A bird's mind-boggling stunt?

- Viewed over 250k times in just 24 hours after a YouTuber uploaded it to the internet.

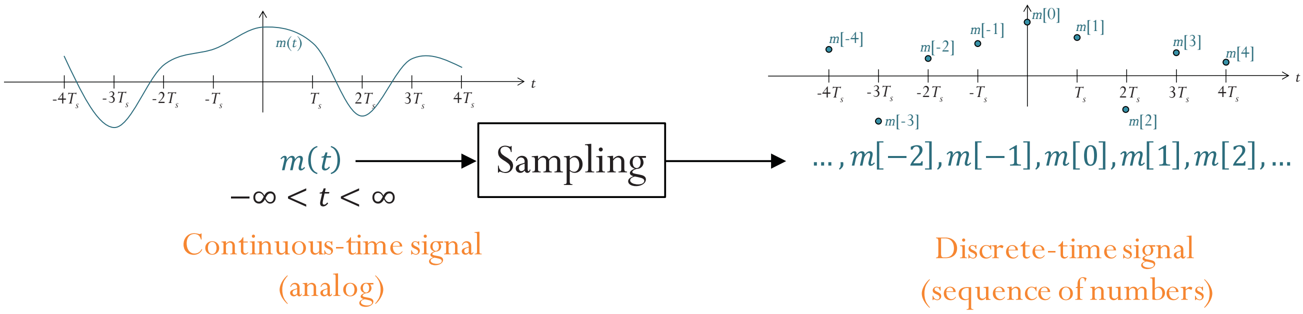


[http://www.boredpanda.com/camera-frame-rate-synced-bird-wings/?utm_source=CB11&utm_medium=link&utm_campaign=SAW]

A magically hovering helicopter?

Sampling

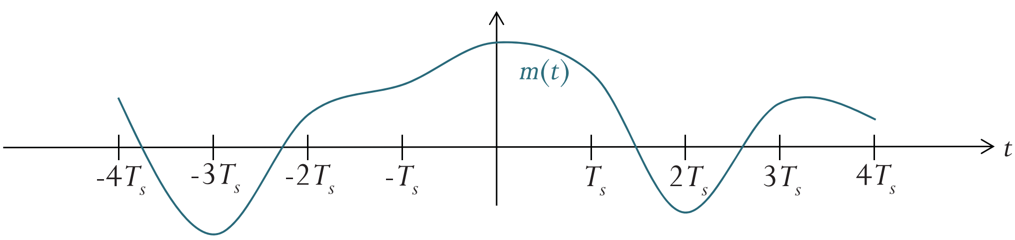
- **Sampling** is the process of taking a (sufficient) number of discrete values of points on a waveform that will define the shape of the wave form.



- Here, the signal is sampled at a uniform rate, once every T_s seconds.
- We refer to $\boldsymbol{T_s}$ as the **sampling period**, and to its reciprocal $\boldsymbol{f_s} = {}^{1}\!/_{T_s}$ as the **sampling rate** which is measured in samples/sec [Sa/s].

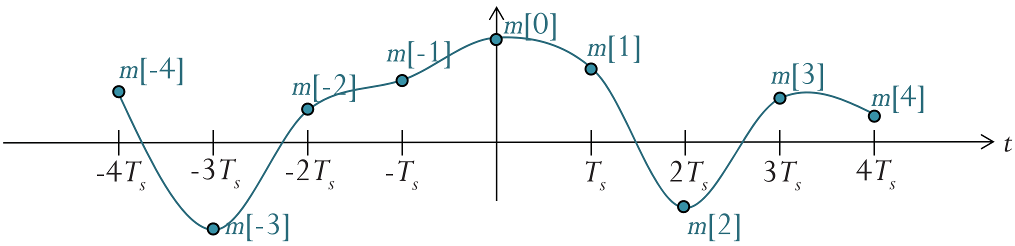
Sampling

- Start with a continuous-time (analog) signal.



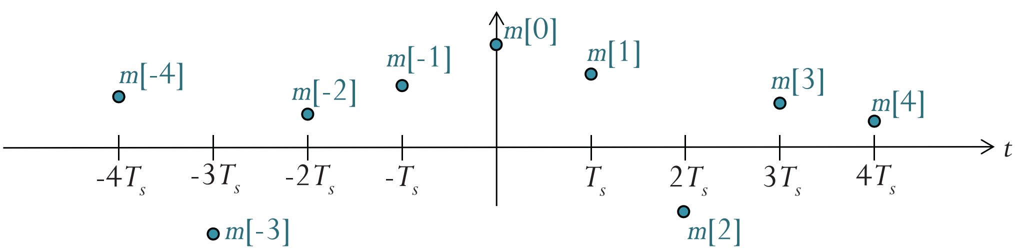
Sampling

- Record the value every T_s seconds.



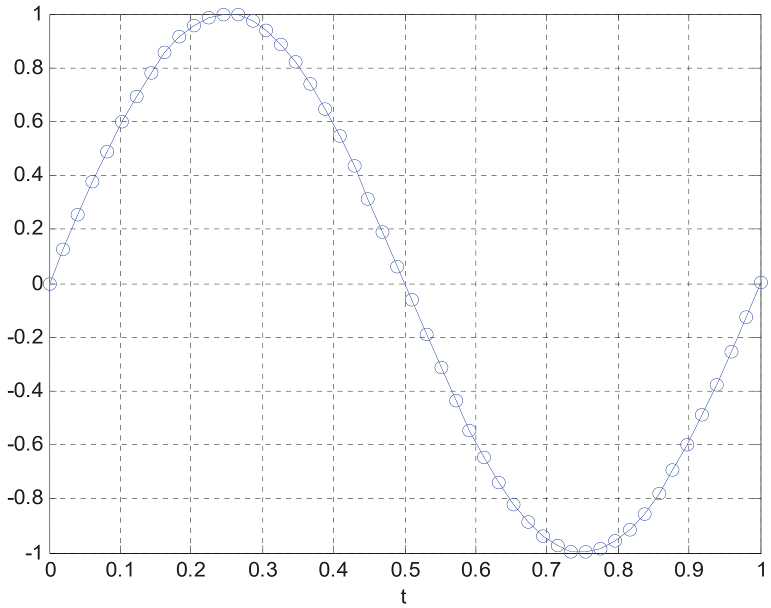
Sampling

- Get a sequence of samples (numbers).



Example: Plotting sin(100πt) (1/6)

This is the plot of sin(100πt). What's wrong with it?

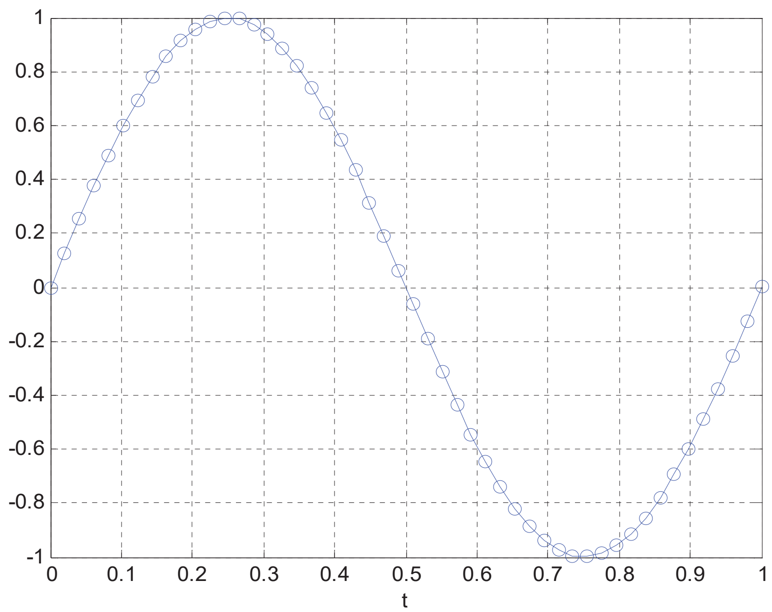


[AliasingSin_FirstEx.m]

Example: Plotting sin(100πt) (2/6)

- Plot 50 points from 0 to 1.

```
close all; clear all;
fs = 49;
ET = 1;
t = 0:1/fs:ET;
x = sin(100*pi*t);
plot(t,x,'-o'); grid on
xlabel('t')
```



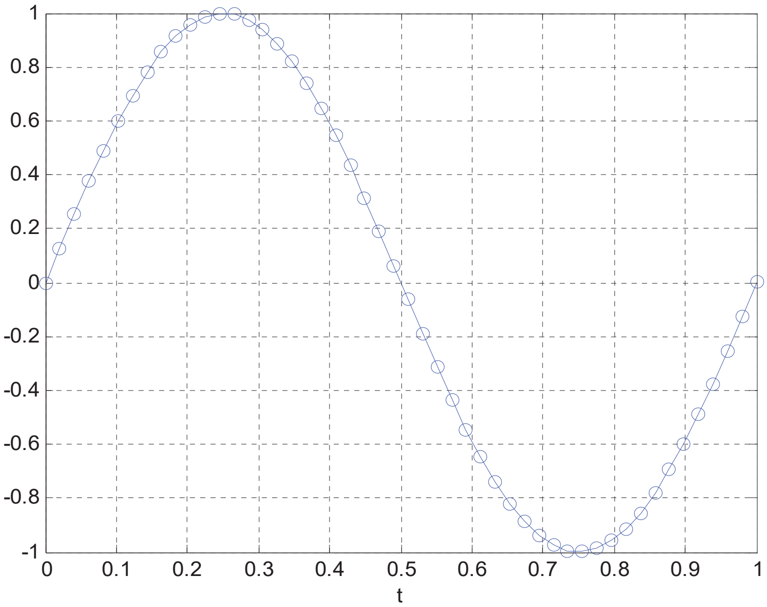
Example: Plotting sin(100πt) (3/6)

Signal of the form $\sin(2\pi f_0 t)$ have frequency $f = f_0$ Hz.

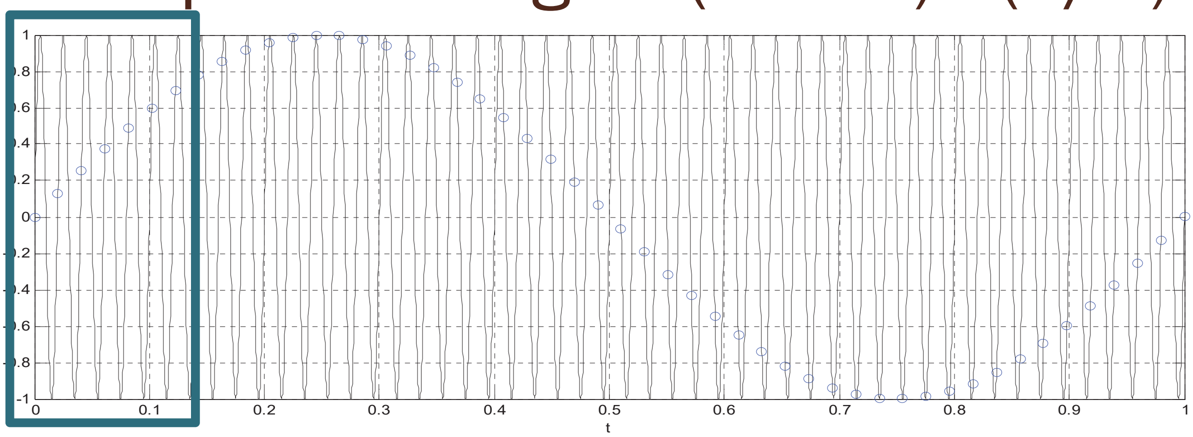
So, the frequency of $\sin(100\pi t)$ is 50 Hz.

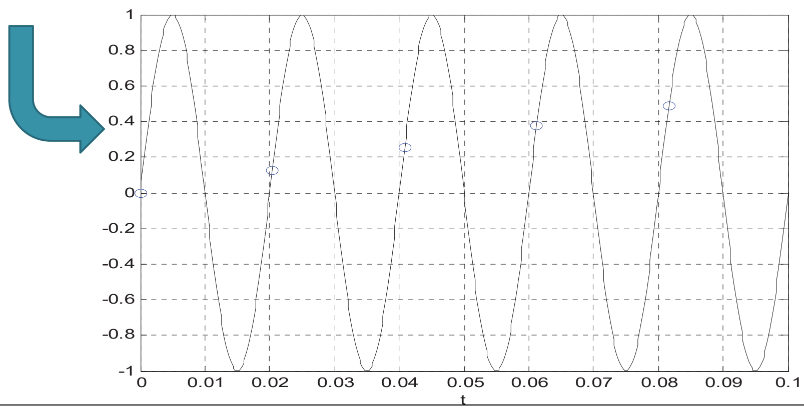
From time 0 to 1, it should have completed 50 cycles. However, our plot has only one cycle.

It looks more like the plot of $\sin(2\pi t)$



Example: Plotting sin(100πt) (4/6)



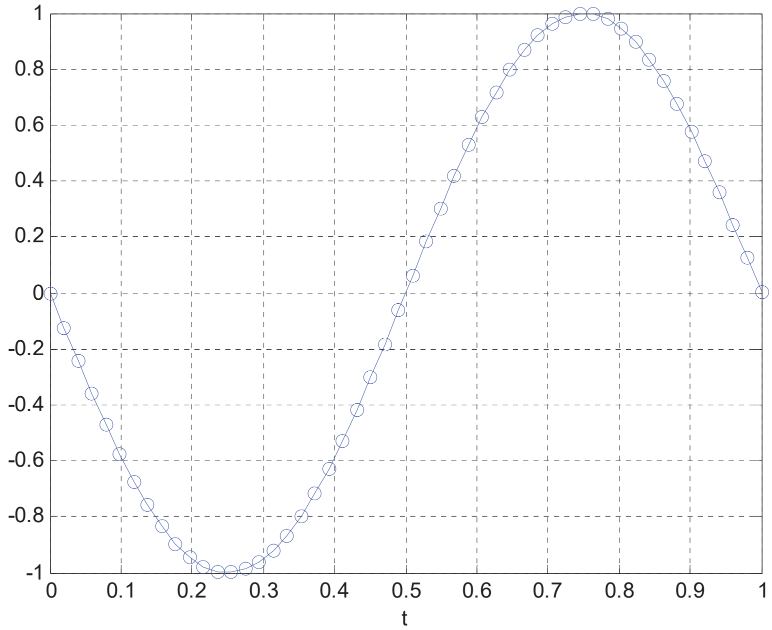


Aliasing causes high-frequency signal to be seen as low frequency.

Example: Plotting sin(100πt) (5/6)

- Plot 52 points from 0 to 1.

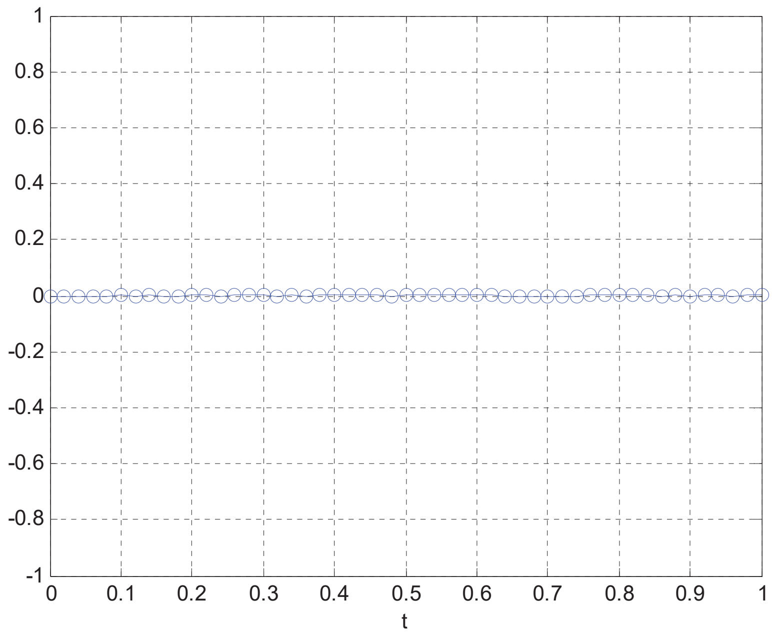
```
close all; clear all;
fs = 51;
ET = 1;
t = 0:1/fs:ET;
x = sin(100*pi*t);
plot(t,x,'-o'); grid on
xlabel('t')
```



Example: Plotting sin(100πt) (6/6)

- Plot 51 points from 0 to 1.

```
close all; clear all;
fs = 50;
ET = 1;
t = 0:1/fs:ET;
x = sin(100*pi*t);
plot(t,x,'-o'); grid on
xlabel('t')
ylim([-1 1])
```



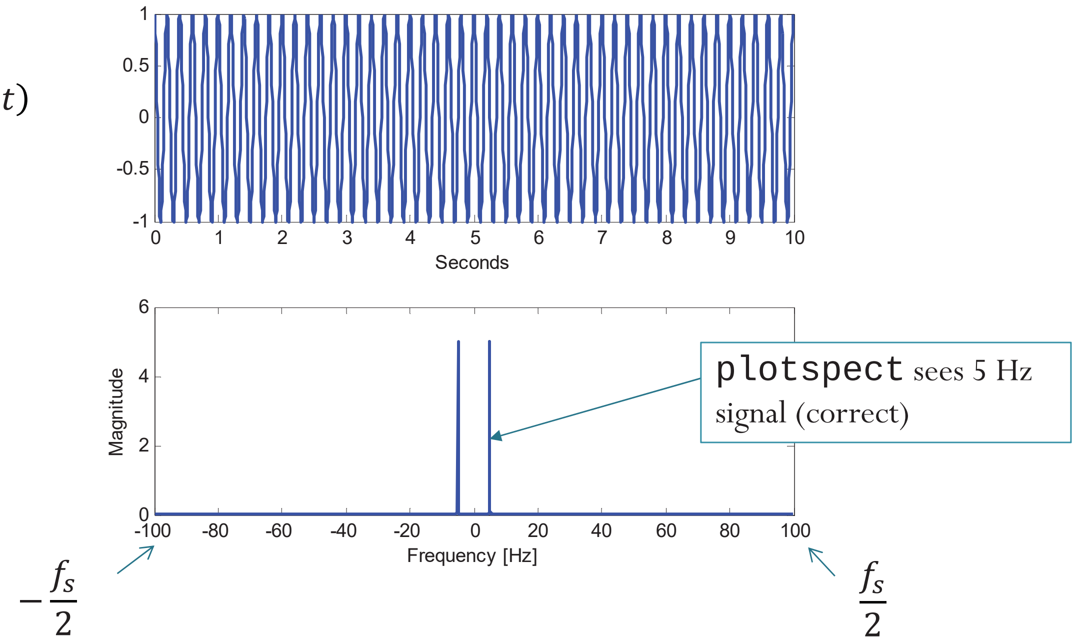
Sampling and Aliasing

- **Sampling Theorem**: In order to (correctly and completely) represent an analog signal, the sampling frequency, f_s , must be at least twice the highest frequency component of the analog signal.
- If the conditions of the sampling theorem are not satisfied, we experience an effect called **aliasing** in which different signals become indistinguishable (or aliases of one another) when sampled.
- The term "aliasing" also refers to the distortion or artifact that results when the signal reconstructed from samples is different from the original continuous signal.

Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

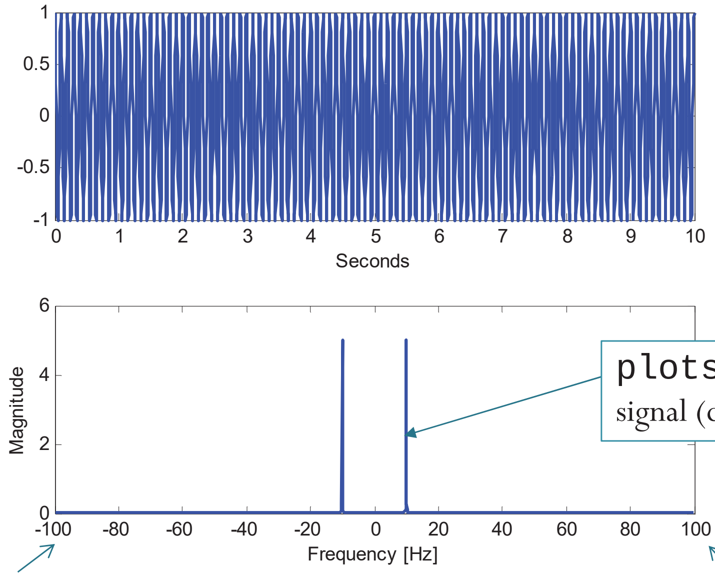
$\cos(2\pi(5)t)$



Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(10)t)$

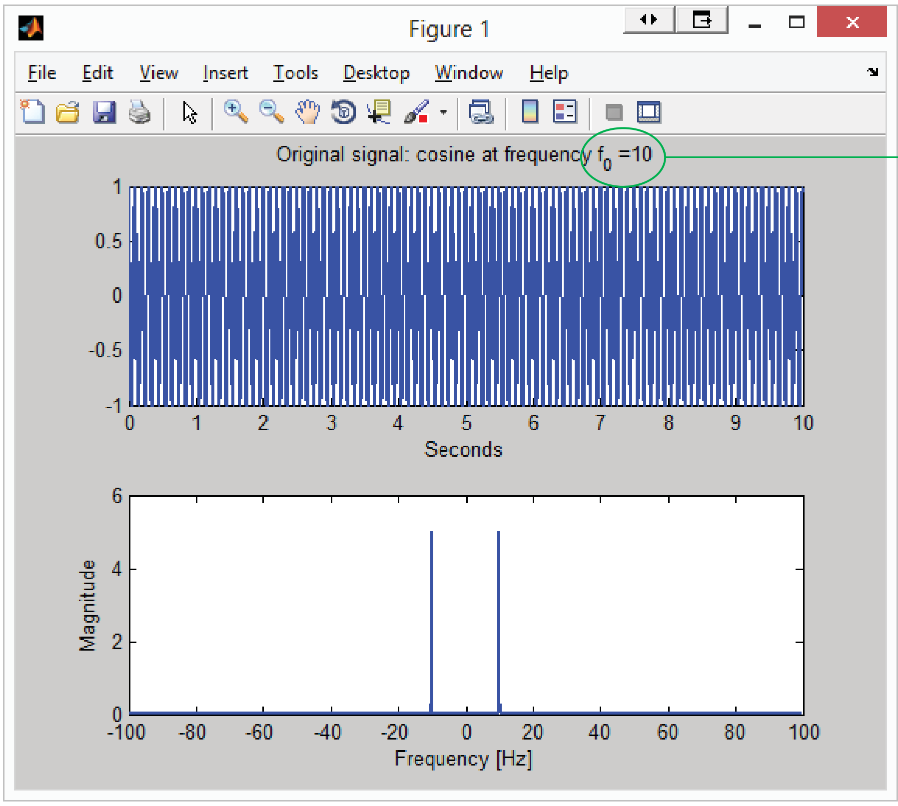


`plotspect` sees 10 Hz signal (correct)

$-\frac{f_s}{2}$ $\frac{f_s}{2}$

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

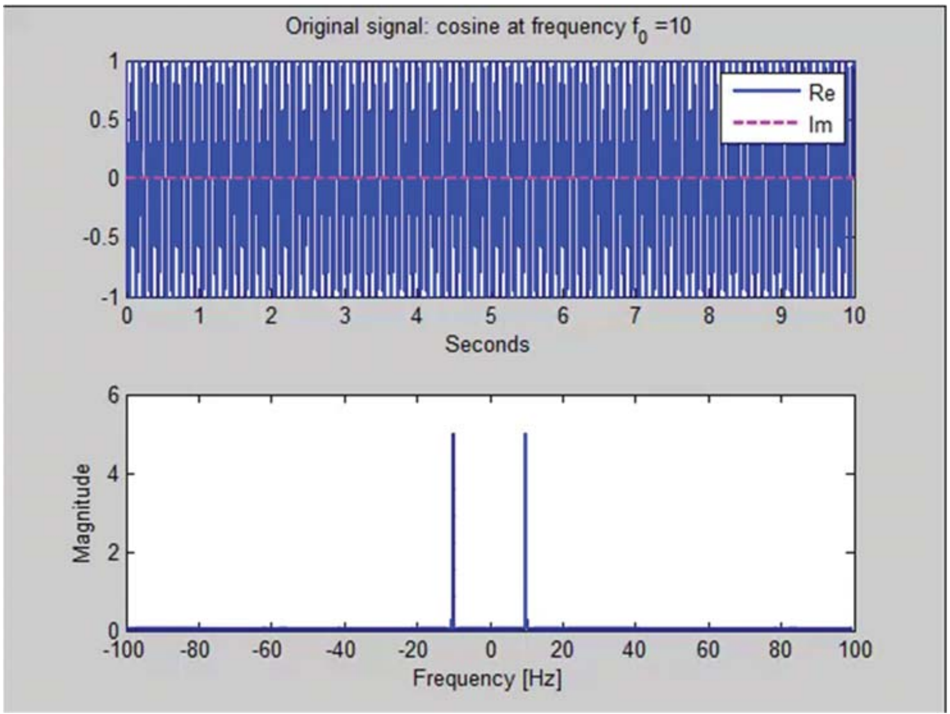


$\cos(2\pi(f_0)t)$

The frequency f_0 of the cosine will be increased (in steps of 10) from 10 Hz to 300 Hz.

[aliasingCos.m]

MATLAB Demo

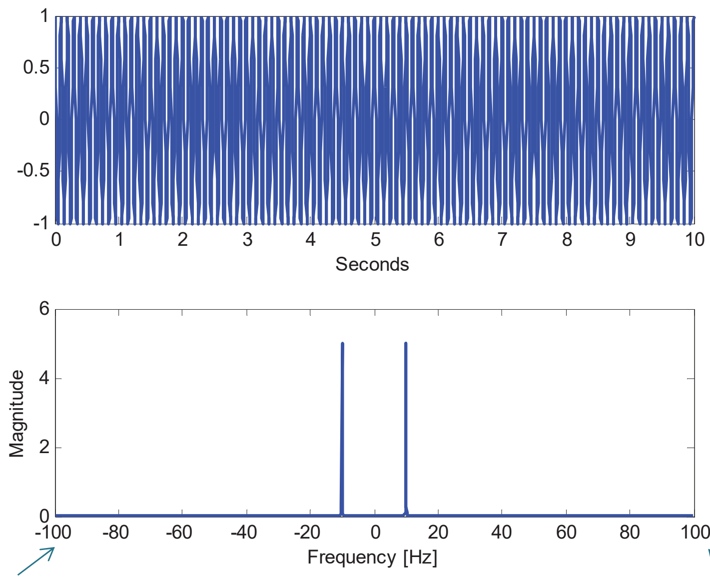


[aliasingCos.m]

Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(10)t)$



$-\frac{f_s}{2}$ $\frac{f_s}{2}$

In the subsequent plots, we will show only the frequency-domain part.

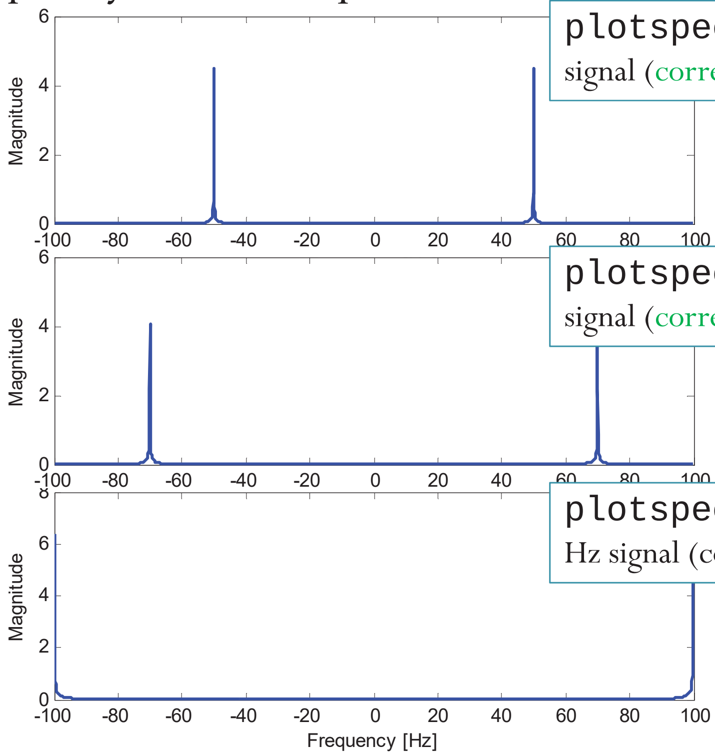
Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(50)t)$

$\cos(2\pi(70)t)$

$\cos(2\pi(100)t)$



`plotspect` sees 50 Hz signal (correct)

`plotspect` sees 70 Hz signal (correct)

`plotspect` sees 100? Hz signal (correct?)

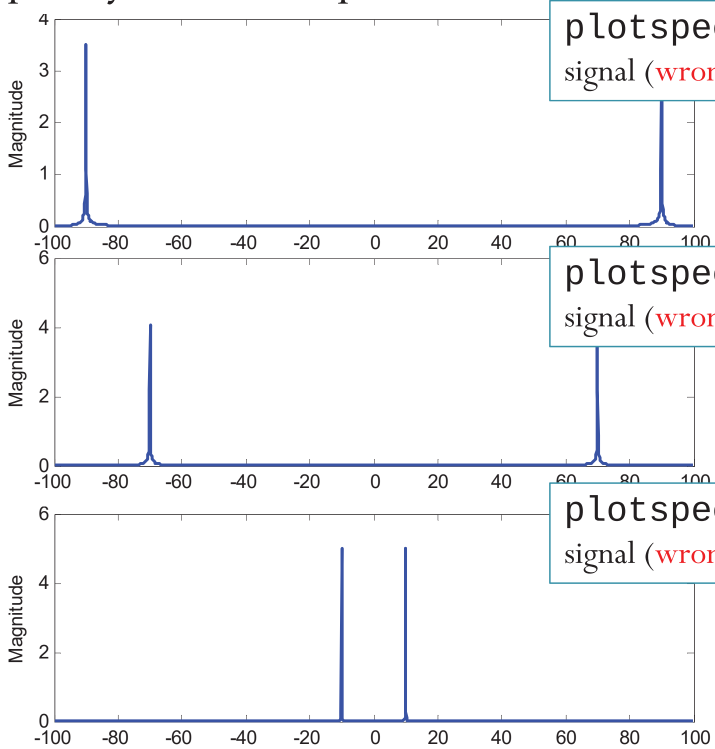
Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(110)t)$

$\cos(2\pi(130)t)$

$\cos(2\pi(190)t)$



`plotspect` sees 90 Hz signal (wrong!)

`plotspect` sees 70 Hz signal (wrong!)

`plotspect` sees 10 Hz signal (wrong!)

Using plotspect.m to study aliasing

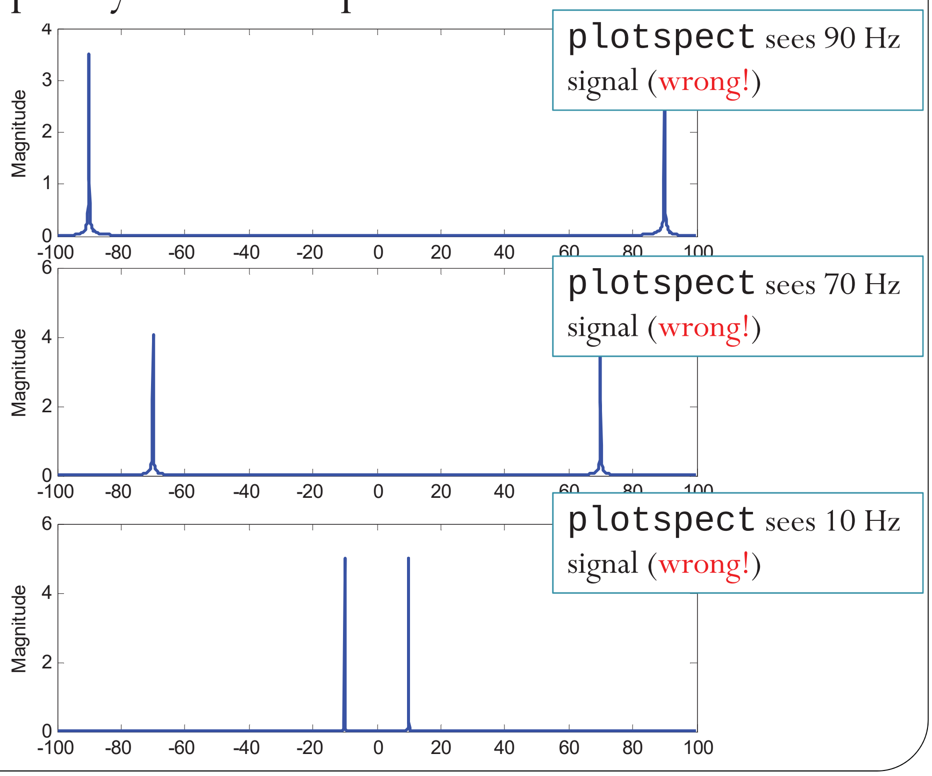
- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(110)t)$

This behavior is commonly referred to as **folding**.

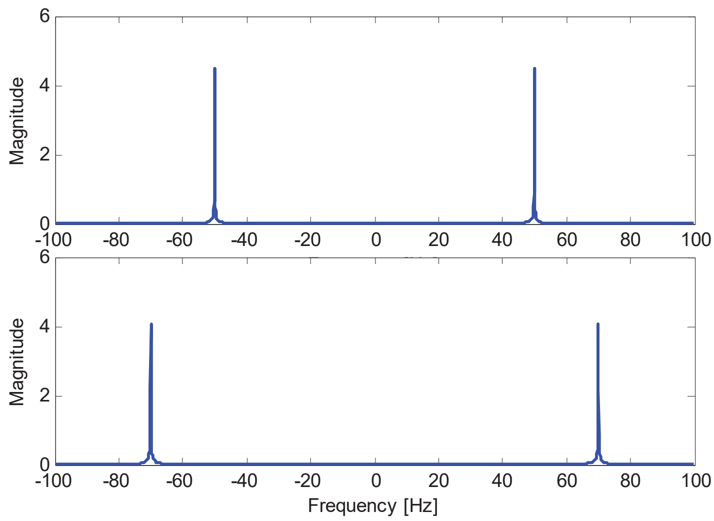
$\cos(2\pi(130)t)$

$\cos(2\pi(190)t)$

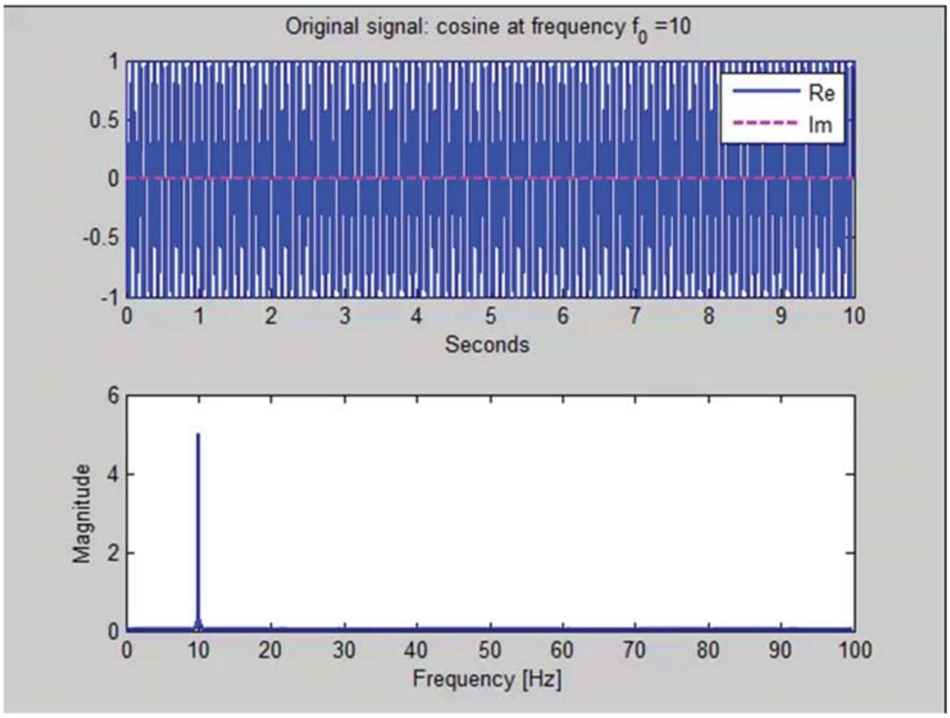


The folding technique

- The even symmetry of the $\cos(2\pi(f_0)t)$ spectrum means that we only have to look at positive frequency to find its perceived frequency



The folding technique

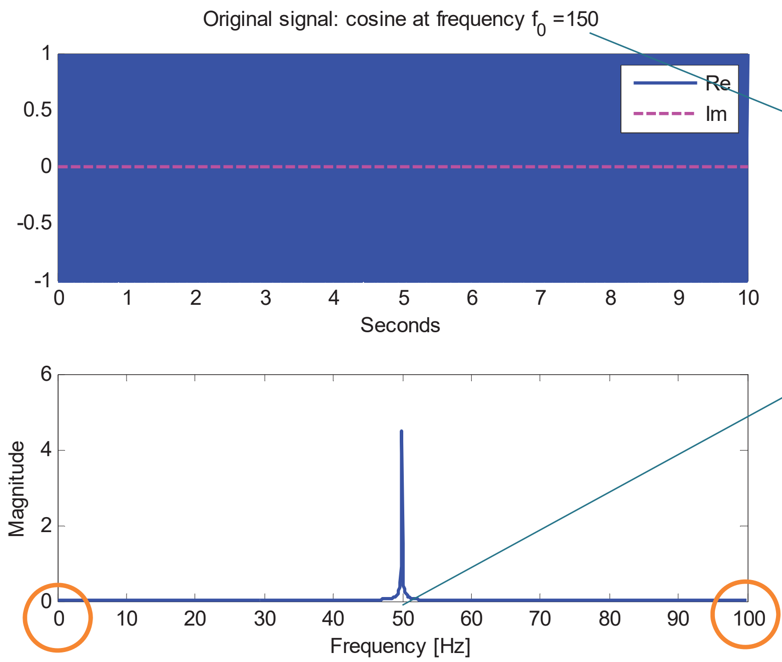


[aliasingCos_folding]

The folding technique

- The folding technique is useful for finding the perceived frequency of $\cos(2\pi(f_0)t)$.

Demo: [aliasingCos_folding]



When $f_s = 200$ [Sa/s], the cosine @ freq. 150 Hz will be perceived as a cosine @ freq. 50 Hz.

Finding the "perceived" frequency

For $\cos(2\pi(f_0)t)$, we may use the "folding technique":

1. Consider the window of frequency from 0 to $\frac{f_s}{2}$.
2. Start from 0, increase the frequency to f_0.
   Fold back at 0 and $\frac{f_s}{2}$ if necessary.

Remark: By the symmetry in the spectrum of cosine, we can always give a nonnegative answer for the perceived frequency.

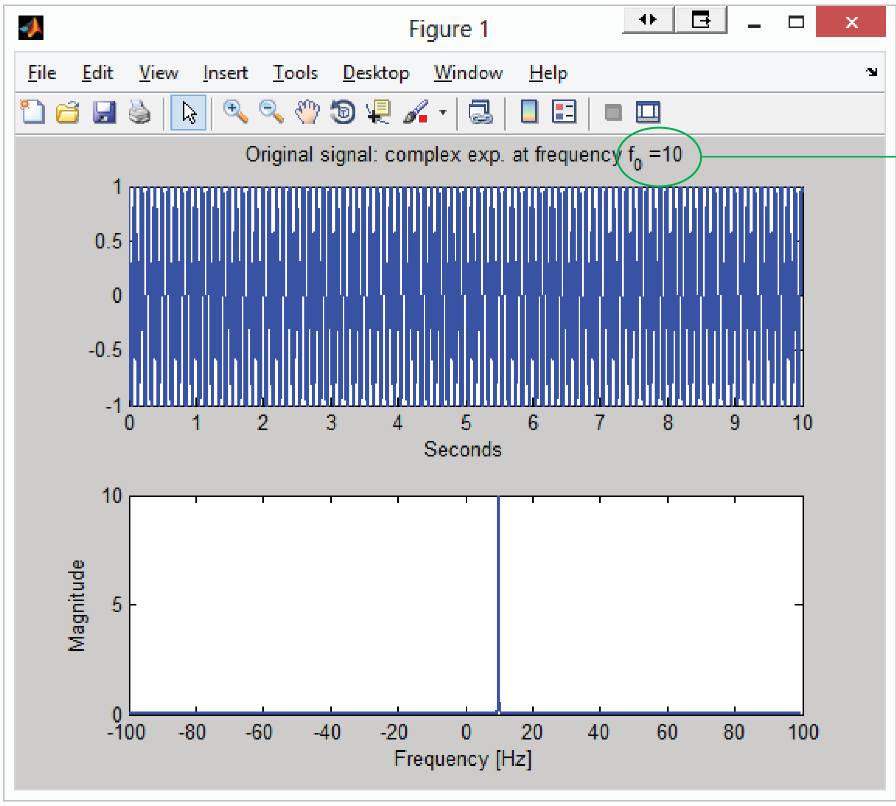
Example: Find the perceived frequency of $\cos(300\pi t)$ when the sampling rate is 200 [Sa/s].

Pac Man's Tunneling

Actually, the delta functions are doing **tunneling** (like in Pac Man).

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

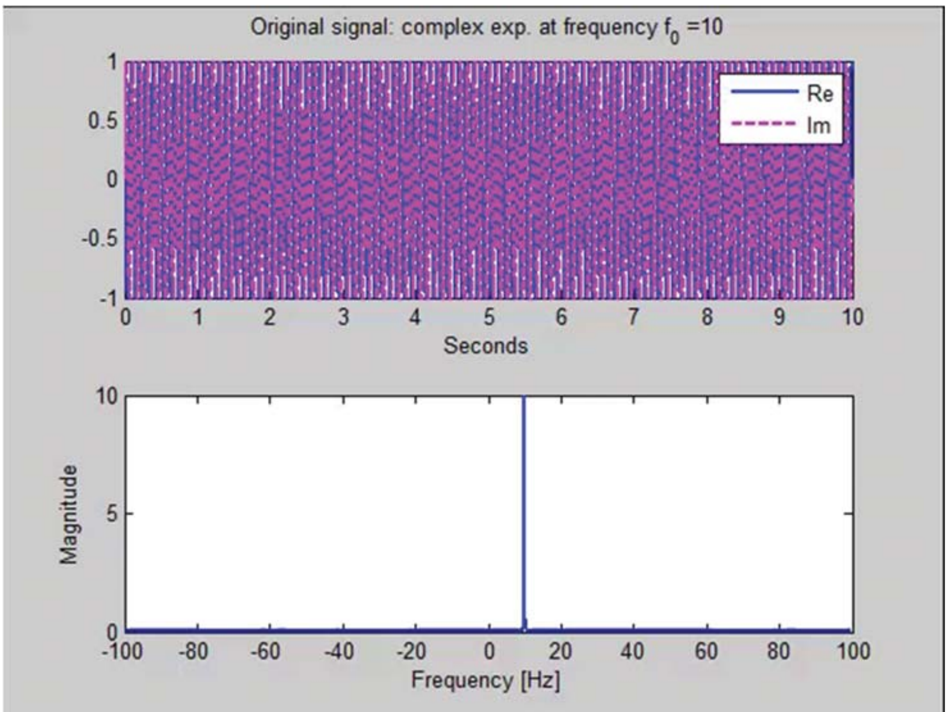


$e^{j2\pi(f_0)t}$

The frequency f_0 of the complex expo. signal is increased (in steps of 10) from 10 Hz to 300 Hz.

[aliasingExp.m]

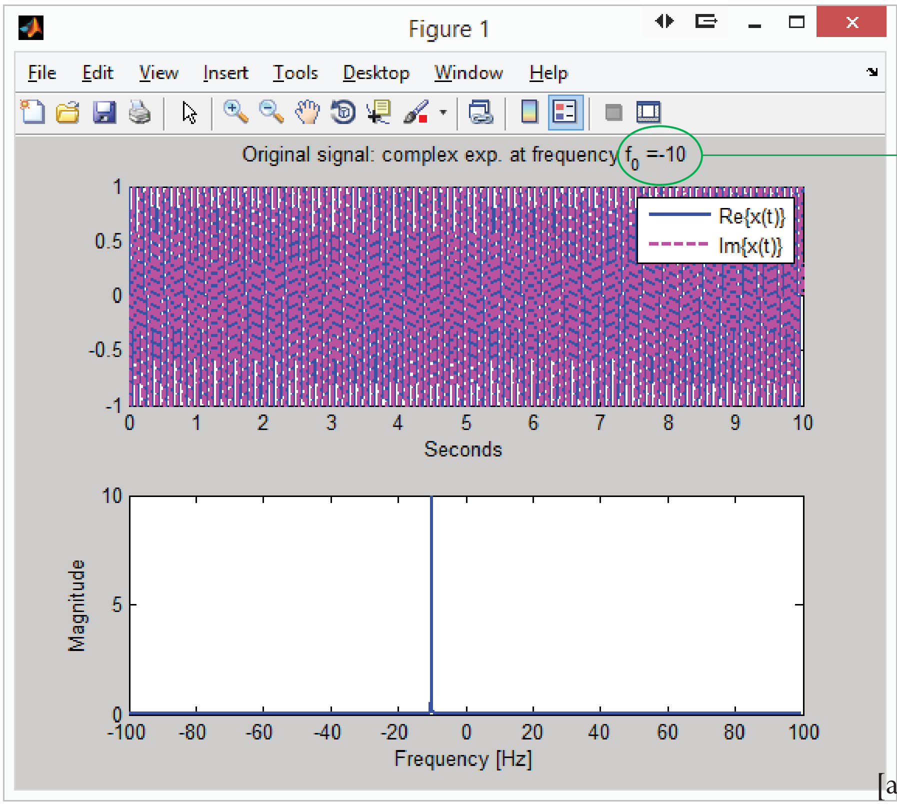
MATLAB Demo



[aliasingExp.m]

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

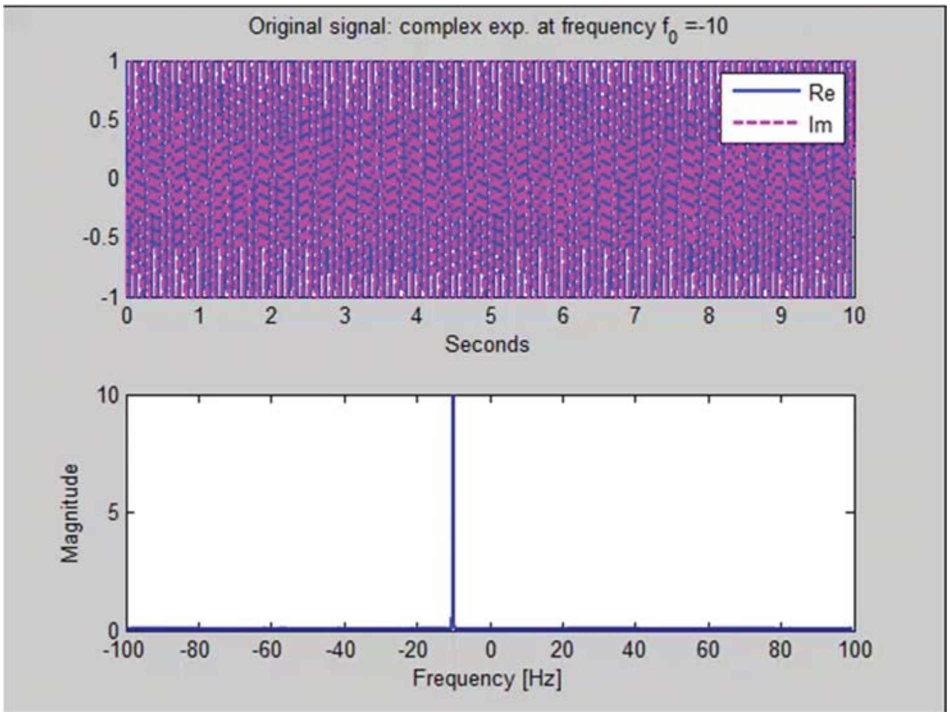


$e^{j2\pi(f_0)t}$

The frequency f_0 of the **complex expo. signal** will be decreased (in steps of 10) from **-10 Hz to -300 Hz.**

[aliasingExpNegative.m]

MATLAB Demo



[aliasingExpNegative.m]

Finding the "perceived" frequency

For $e^{j2\pi(f_0)t}$, we use the "tunneling technique":

1. Consider the window of frequency from $-\frac{f_s}{2}$ to $\frac{f_s}{2}$.
2. Start from 0.
   a) If $f_0 > 0$, increase the frequency to f_0 (going to the right). Restart at $-\frac{f_s}{2}$ when $\frac{f_s}{2}$ is reached.
   b) If $f_0 < 0$, decrease the frequency to f_0 (going to the left). Restart at $+\frac{f_s}{2}$ when $-\frac{f_s}{2}$ is reached.

Example: Find the perceived frequency of $e^{j300\pi t}$ when the sampling rate is 200 [Sa/s].